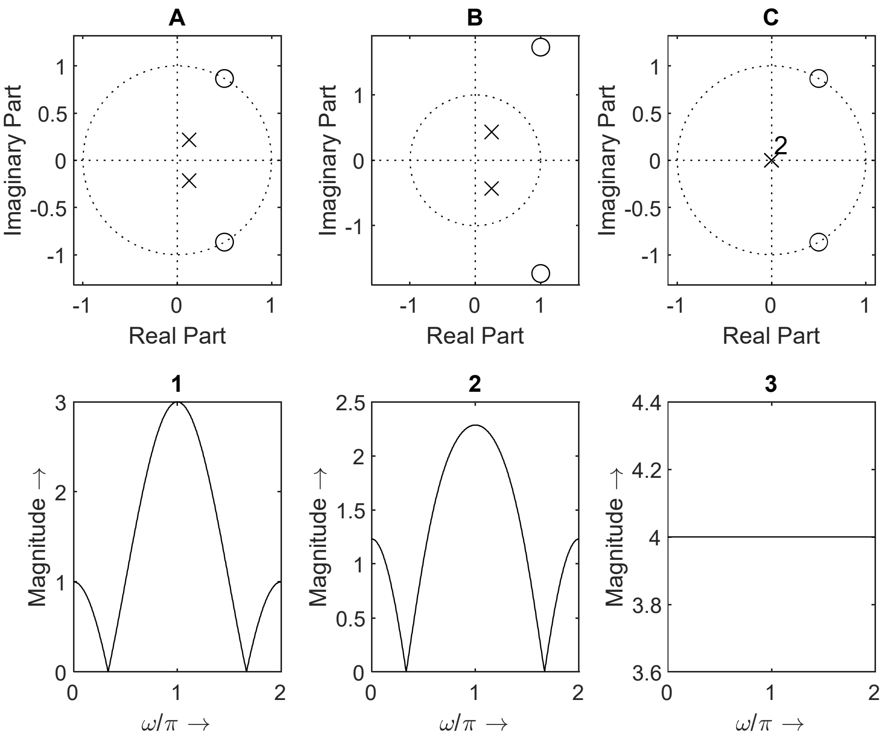
L.EEC/M.EEC
L.EEC025 - Fundamentals of Signal Processing

FIRST EXAM, JANUARY 08, 2025
Duration: 120 Minutes, closed book

NOTE: each question ***must*** be answered in a separate sheet; please write your name and Student order number on all sheets, please provide complete answers while trying to minimize paper usage. Except for a basic scientific calculator and the provided formulae sheet, no other materials or tools, including (so-called) AI assistants, are allowed in this exam.
DISCLAIMER: no FunSP materials, including this exam, have been produced using any (so-called) AI-based tools.

1. Three different causal discrete-time systems have zero-pole diagrams A, B, and C, and the frequency response magnitudes 1, 2, and 3, as represented next. The radius of each represented pole, or zero, is either 0.25, 0.5, 1.0 or 2.0, and the angle of each represented pole, or zero, is either +π/3 rad or -π/3 rad.



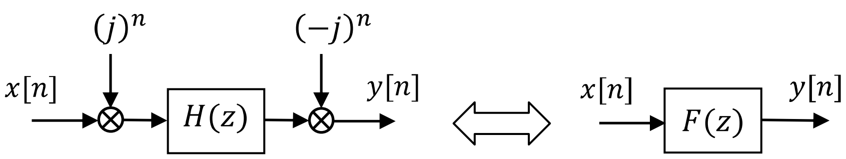
a) **[1,5 *pts*]** Match each zero-pole diagram (A, B, C) to the corresponding frequency response magnitude (1, 2, 3), and indicate the main supporting arguments.

b) **[1 *pt*]** Which of the represented systems (A, B, or C) are stable ? And which ones have a real-valued impulse response ? Why ?

c) **[0.5 *pts*]** What practical applications are the represented systems good for ? Give at least two examples.

d) **[1,5 *pts*]** Consider the following statement: «If $h_A[n]$ and $h_B[n]$ represent the impulse response of system A and system B, respectively, then $h_B[n]$ may be obtained by modifying $h_A[n]$ using one single and simple transformation». Is it true or false ? If true, what is that single and simple transformation ?

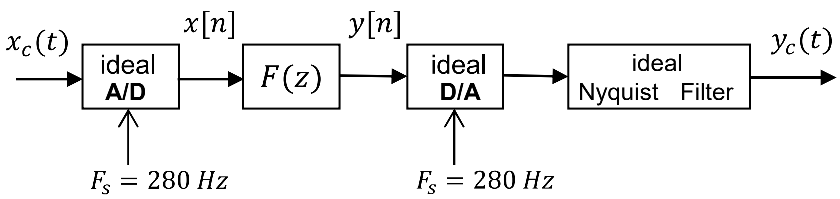
2. Consider the illustrated system equivalence.



a) **[1 *pt*]** Show that $F(z) = H(jz)$.

b) **[1,5 *pts*]** Admit that $H(z) = (1 - z^{-1})(1 + z^{-1})$. Obtain a compact expression describing the frequency response of the overall system, $F(e^{j\omega})$, as well as its magnitude and phase parts in the range $[0, 2\pi[$, which you should sketch.

c) **[1 *pt*]** Consider the illustrated analog and causal discrete-time system whose transfer function is $F(z)$, as suggested in **a)**. An *anti-aliasing* filter does not exist and the input analog signal is $x_c(t) = 1 + \sin(490\pi t) + \cos(420\pi t)$.



Find the sinusoidal frequencies of the discrete-time signal $x[n]$ in the Nyquist range, i.e., in the range $-\pi \le \omega < \pi$. Obtain a compact expression for $x[n]$.

d) **[1,5 *pts*]** Obtain $y[n]$ and, presuming ideal reconstruction conditions, obtain $y_c(t)$.

3. A causal discrete-time system is characterized by the following difference equation: $y[n] = x[n] - x[n-1] + 0.1y[n-1] + 0.2y[n-2]$.

a) **[1 *pt*]** Obtain the transfer function of that system (including the RoC), sketch a possible canonic realization structure, and indicate what its implementation cost is in terms of arithmetic operations and memory.

b) **[1,5 *pts*]** Determine the output sequence when the input sequence exciting the system is $x[n] = u[n]$.

c) **[1 *pt*]** Admit that each unitary delay (z^{-1}) in the delay chain of the realization structure of the system is replaced by z^{-2}. What is the impact of this modification to the order of the system, to its frequency response magnitude, and to the poles and zeros of the new system admitting that the poles and zeros of the original (unmodified) system are all real-valued ?

4. Consider the following Matlab code.

```
x=[1 0 3 0 5 0];
X=fft(x); N=length(X);
Y=zeros(size(X)); Y(1)=X(1);
Y(N:-1:2)=X(2:N);
Z=X.*Y; ifft(Z)
k=[0:N-1]; W=X.*(1+cos(k*2*pi/N)); ifft(W)
```

a) **[1 *pt*]** Express $z[n]$ as a function of $x[n]$. Find the result of `ifft(Z)` without executing the code.

b) **[1,5 *pts*]** Express $w[n]$ as a function of $x[n]$.

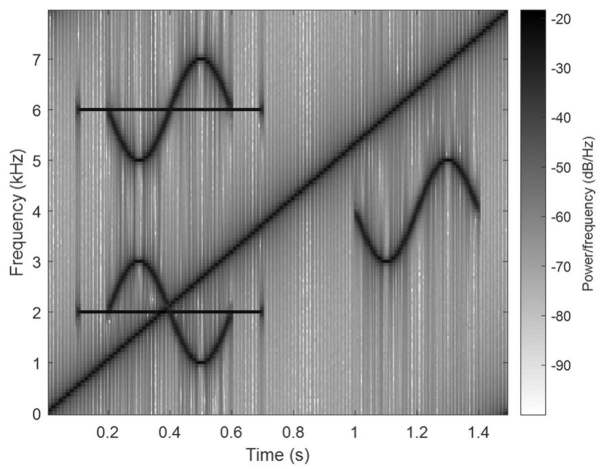
Hint: Express first the `cos()` function as a function of complex exponentials.

(**continuation of Exercise 4**)

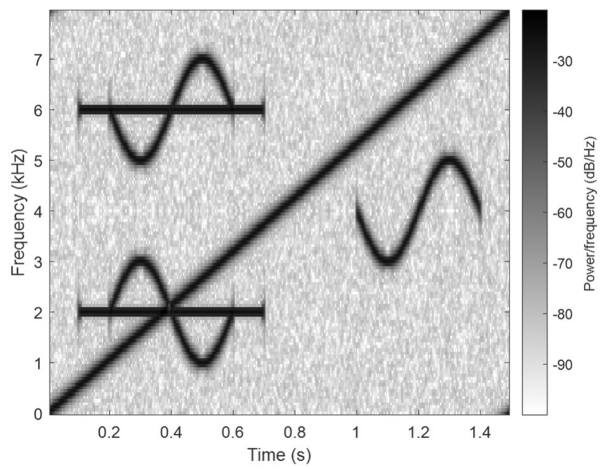
c) **[1 *pt*]** Find the result of `ifft(W)` without executing the code.

5. The spectral contents of a synthetic signal sampled at F_S=8000 samples/s was analyzed by means of spectrograms. The signal contains both real-valued and complex-valued signal components. Admit that their starting frequencies are within the Nyquist range, and that they evolve *monotonously* through time. The two spectrograms A and B represented next were obtained using two alternative windows (Hanning and Rectangular), and a sliding FFT (N=128), with 50% overlap between adjacent FFTs.

A



B



a) **[1 *pt*]** How many real-valued signal components does the signal contain ? And how many complex-valued signal components ? What window has been used to generate spectrogram A ? What window has been used to generate spectrogram B ? Explain your reasoning.

Note: the blurred effects in the spectrograms reflect the impact of signal processing, and not printer problems.

b) **[1 *pt*]** Represent two plausible periodograms (or power spectra), one corresponding to `t=0.5 s`, and another to `t=1.3 s`. Explain.

c) **[1,5 *pts*]** Based on the observation of the spectrograms, describe the spectral contents of the synthetic signal.

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