L.EEC025 - FUNDAMENTALS OF SIGNAL PROCESSING

Academic year 2024-2025, week 12
P2P exercises

Topics: "Peer-to-peer learning/assessment" exercises addressing the circular properties of the DFT, and sinusoidal frequency estimation using the DFT.

Exercises related to "Peer-to-peer learning/assessment" (P2P L/A)

P2P Exercise 1

Consider the following Matlab code:

```
N=5;
a=[2  2-j  1-2j  2j  1];
b=[-j  -j  j  j  -3j];
A=fft(a); B=fft(b);
X(1)=j*imag(A(1)); X(2:N)=0.5*(A(2:N)-conj(A(N:-1:2)));
Y= -j*imag(B);
Z=Y.*X; z=ifft(Z)
```

a) Explain to your colleagues how you can find the result of the Matlab command `ifft(X)` without computing any DFT or IDFT, i.e. by just using the properties of the DFT. Show that result to your colleagues.

b) Explain how you can find the result of the Matlab command `ifft(Y)` without computing any DFT or IDFT, i.e. by just using the properties of the DFT. Show to your colleagues how you obtain that result.

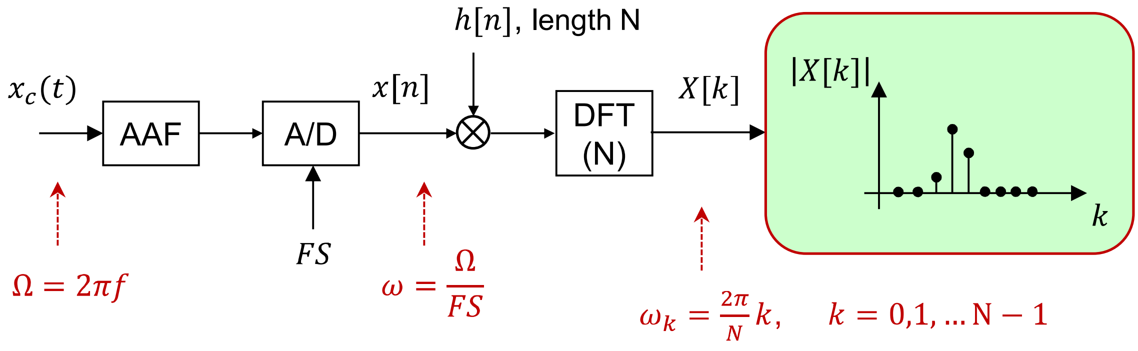
Note: the solution should be: `[j  2j  -j  -j  2j]`

c) Without computing any DFT or IDFT, demonstrate how you can obtain the result of `z=ifft(Z)`

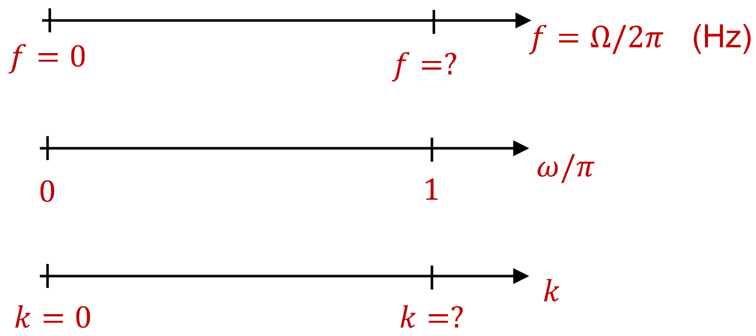
Note: the solution should be: `[2  7  0  1  -7]`

P2P Exercise 2

In this exercise, we use the DFT in order to estimate the frequency f of a continuous-time sinusoid $x_c(t) = \sin(2\pi f t)$. This analog sinusoidal signal is (ideally) sampled (the sampling frequency is FS) and then a segment of length N of the discrete-time signal, $x[n]$, is multiplied by a window $h[n]$ having also length N. As illustrated in the following block diagram, the signal is then DFT transformed and the resulting discrete-frequency sequence, $X[k]$, is analyzed. A local maximum in the magnitude spectrum $|X[k]|$ denotes the frequency of the sinusoid. The purpose of this exercise is to demonstrate this possibility and to verify the quality of the frequency estimation as a function of f, $h[n]$, and N.



a) In different points of the above signal processing chain, frequency is looked at in different perspectives, i.e. according to different axes as the following scheme illustrates. Explain to your colleagues the correspondence between the three axes (two of them continuous, one of them discrete) and replace the question marks by the correct values (assume that $FS = 8000\ Hz$).



b) The frequency resolution of the DFT is defined as the frequency separation between two consecutive DFT lines (also referred to as DFT bins). If $FS = 8000\ Hz$, and if $N = 64$, what is the DFT frequency resolution in Hertz ?

c) If $h[n] = u[n] - u[n-N]$, find $|H(e^{j\omega})|$.

Note: the solution should be:

$$|H(e^{j\omega})| = N \left| \frac{\mathrm{sinc}(\omega N/2)}{\mathrm{sinc}(\omega/2)} \right|$$

Note 2: in this definition, $\mathrm{sinc}(\theta) = \frac{\sin(\theta)}{\theta}$, however, Matlab considers $\mathrm{sinc}(\theta) = \frac{\sin(\pi\theta)}{\pi\theta}$.

d) If $x_c(t) = \sin(2\pi f t)$, find what the Fourier Transform is of the product $x[n]h[n]$, where $h[n] = u[n] - u[n-N]$; we admit that $x[n]$ is an infinite-length sinusoidal signal.

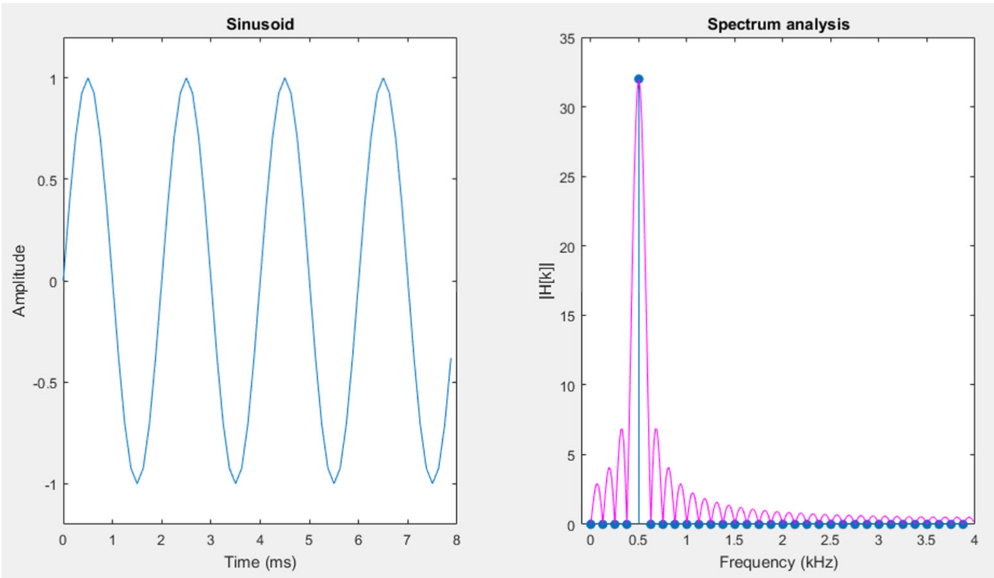
Note: to simplify, you may represent and explain what the absolute value of that Fourier Transform is.

Hint: recall that product in the discrete-time domain means convolution in the frequency domain.

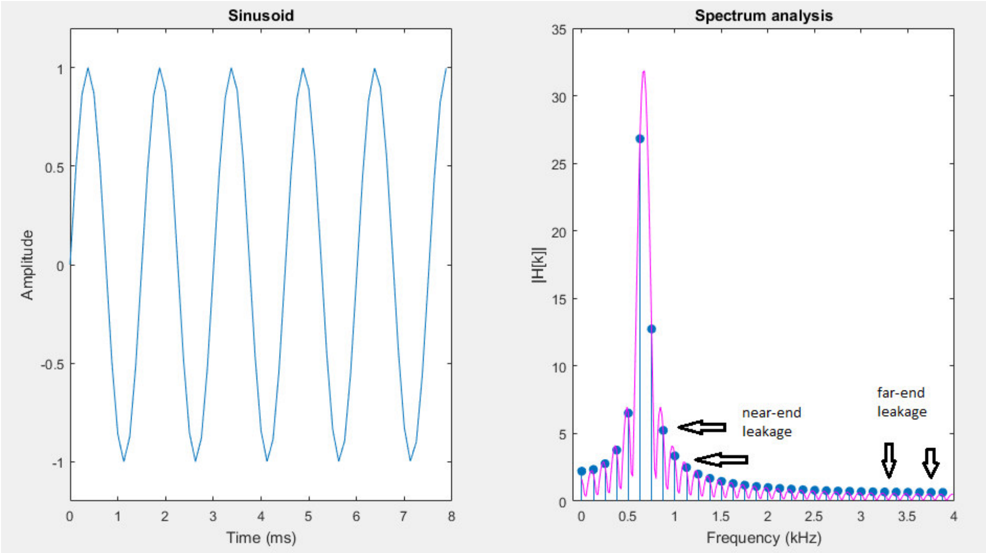
e) Admit that $FS = 8000$ Hz. Use the Matlab command file named `FunSP_sinus_spectrum.m` that is available on Moodle in order to illustrate the magnitude of the Fourier Transform discussed in **c)**, as well as $|X[k]|$, when f varies between 500 Hz and 1600 Hz, in steps of 2.5 Hz.

Explain the structure of the Matlab code to your colleagues, and run the Matlab code for a few iterations by hitting "Enter" several times while showing to your colleagues all graphical outputs.

In particular, explain why only one DFT spectral line is different from zero, as in the following illustrative example,



or, when all DFT lines are different from zero, as in the following illustration.



f) When an exact integer number of periods of the sampled sinusoidal wave fits within the $h[n]$ window, as demonstrated in **e)**, then $|X[k]|$ consists of a single non-zero spectral line, why ?

g) The `FunSP_sinus_spectrum.m` Matlab code also finds and represents the frequency estimation error (i.e. the frequency estimation error relative to the DFT frequency resolution) of a crude frequency estimator. Explain to your colleagues what this frequency estimation error is and why is that the error varies between 0% and around 50%.

h) **OPTIONAL**: Just for illustration purposes, the `FunSP_sinus_spectrum.m` Matlab code also includes an improved frequency estimation algorithm (which you do not need to explain) that reduces immensely the frequency estimation error when compared to the crude frequency estimator. You can show to your colleagues its operation by uncommenting the following Matlab command:

```
% estimatedfreq=(ell-1+delta)*resolution;
```