PDSILAB10

2nd part: problems 5 and 6

$\boxed{5}$ $x[n] = 2^{-n}u[n] = 0.5^{n}u[n]$

a) $X(e^{j\omega}) = \dfrac{1}{1-0.5e^{-j\omega}}$

b) $\underset{\text{(aperiodic)}}{x[n]} \xrightarrow{\mathcal{F}} \underset{\text{(2π-periodic)}}{X(e^{j\omega})} \xrightarrow{\text{sampling}} \underset{\text{(N-periodic)}}{Y[k]} = X(e^{j\omega})\Big|_{\omega = k\frac{2\pi}{N}}$

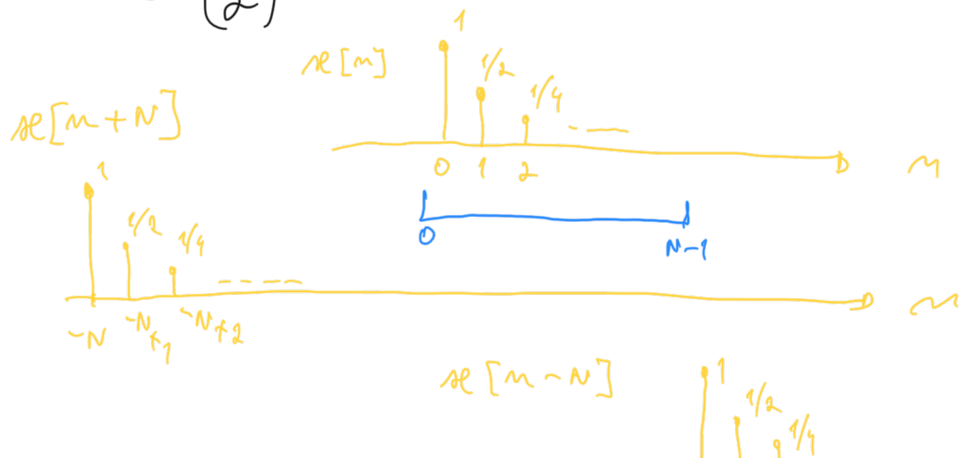
k=0,1,...,N-1

$$\underset{\text{(N-periodic)}}{Y[k]} \xrightarrow{DFT^{-1}} \underset{\text{(N-periodic)}}{y[n]} = f\{x[n]\} = \sum_{l=-\infty}^{+\infty} x[n-lN] = \frac{2^{-n}}{1-2^{-N}}$$

$n = 0, 1, \dots, N-1$

$$= \dots + x[n+N] \leftarrow ✓$$
$$+\ x[n] \leftarrow ✓$$
$$+\ x[n-N] \leftarrow ✗$$
$$+\ \dots$$

$x[n] = \left(\frac{1}{2}\right)^{n}u[n]$



$$y[n] = \sum_{l=-\infty}^{0} x[n-lN] = \sum_{l=-\infty}^{0} \left(\frac{1}{2}\right)^{(n-lN)} =$$

$$x[n] = \left(\frac{1}{2}\right)^{n} u[n]$$

$$y[n] = \sum_{l=-\infty}^{0} \left(\frac{1}{2}\right)^{n} \left(\frac{1}{2}\right)^{-lN} = \left(\frac{1}{2}\right)^{n} \sum_{l=-\infty}^{0} \left(\frac{1}{2}\right)^{-lN}$$

$$= \left(\frac{1}{2}\right)^{n} \sum_{l=0}^{+\infty} \left(\frac{1}{2}\right)^{lN} = \left(\frac{1}{2}\right)^{n} \sum_{l=0}^{+\infty} \left(2^{-N}\right)^{l}$$

$$= \left(\frac{1}{2}\right)^{n} \frac{1-0}{1-2^{-N}} = \frac{\left(\frac{1}{2}\right)^{n}}{1-2^{-N}} = \frac{2^{-n}}{1-2^{-N}}$$

$\boxed{6}$

$$x_0[n] = [0 \;\; 1 \;\; 1 \;\; 0]$$

$$x[n] = [0 \;\; 0 \;\; 0 \;\; 0 \;\; \overbrace{0 \;\; 1 \;\; 1 \;\; 0}^{x_0[n]}]$$

a) $x_0[n] \xleftrightarrow{DFT} X_0[k] = \sum_{l=0}^{N-1} x_0[l]\, W_N^{lk}$

(N-periodic) (N-periodic)

b) $X[k] = f\left\{ X_0[k] \right\}$

$x[n] \xleftrightarrow{DFT} X[k]$

(2N-periodic) (2N-periodic)

$2N-1$

$$X[k] \overset{\Delta}{=} \sum_{n=0} x[n] W_{2N}^{kn}$$

$$= \sum_{n=0}^{N-1} \underbrace{x[n]}_{0} W_{2N}^{kn} + \sum_{n=0}^{N-1} \underbrace{x[n+N]}_{= x_0[n]} W_{2N}^{k(n+N)}$$

$$\underbrace{\qquad\qquad\qquad}_{=0}$$

$$= \sum_{n=0}^{N-1} x_0[n] W_{2N}^{kn} W_{\cancel{2N}}^{k\cancel{N}} = e^{-j k \frac{\cancel{2}\pi}{\cancel{2}}} = \left(e^{-j\pi}\right)^k = (-1)^k$$

$$= (-1)^k \sum_{n=0}^{N-1} x_0[n] W_{2N}^{kn}$$

$$\boxed{X[2k]} = \underbrace{(-1)^{2k}}_{=1} \sum_{n=0}^{N-1} x_0[n] W_{\cancel{2}N}^{\cancel{2}kn}$$

$$= \sum_{n=0}^{N-1} x_0[n] W_N^{kn} = \boxed{X_0[k]}$$

$$\boxed{X[2k+1]} = \underbrace{(-1)^{2k+1}}_{=-1} \sum_{n=0}^{N-1} x_0[n] \underbrace{W_{2N}^{(2k+1)n}}_{= W_N^{(k+\frac{1}{2})n}}$$

$$= -\sum_{n=0}^{N-1} x_0[n] W_N^{(k+\frac{1}{2})n}$$

$$x_0[n] = \frac{1}{N} \sum_{l=0}^{N-1} X_0[l] W_N^{-ln}$$

$N-1$ $N-1$ $-ln$ $(k+\frac{1}{2})n$

$$= - \sum_{n=0} \frac{1}{N} \sum_{l=0} X_0[l] \, W_N^{-ln} \, W_N^{(k+\frac{1}{2})n}$$

$$= - \frac{1}{N} \sum_{l=0}^{N-1} X_0[l] \underbrace{\sum_{n=0}^{N-1} W_N^{(k+\frac{1}{2}-l)n}}$$

$$= \frac{1 - W_N^{(k+\frac{1}{2}-l)N}}{1 - W_N^{k+\frac{1}{2}-l}} \qquad = W^{1/2} = -1$$

$$= \frac{2}{1 - W_N^{k+\frac{1}{2}-l}}$$

$$= \boxed{\frac{2}{N} \sum_{l=0}^{N-1} \frac{X_0[l]}{1 - W_N^{k+\frac{1}{2}-l}}}$$