L.EEC025 - FUNDAMENTALS OF SIGNAL PROCESSING

Academic year 2024-2025, week 10
P2P exercises

Topic: Design of a linear-phase FIR differentiator and Hilbert Transformer.

Exercises related to "Peer-to-peer learning/assessment" (P2P L/A)

NOTE: this week (November 25), Student "E3" of each group should explain exercise P2P Exercise 1, and Student "E4" of each group should explain P2P Exercise 2. Detailed information on the P2P procedure is available on Moodle ("Instructions regarding "Peer-to-peer learning/assessment" (P2P L/A)").

P2P Exercise 1

In this exercise, we design a linear-phase FIR differentiator of order 80 (this means that the length of the impulse response is 81 samples) whose ideal 2π–periodic frequency response is specified as follows:

$$H\left(e^{j\omega}\right) = j\omega, \quad -\pi \leq \omega < \pi$$

a) Find the impulse response and explain to your colleagues how you obtain it.

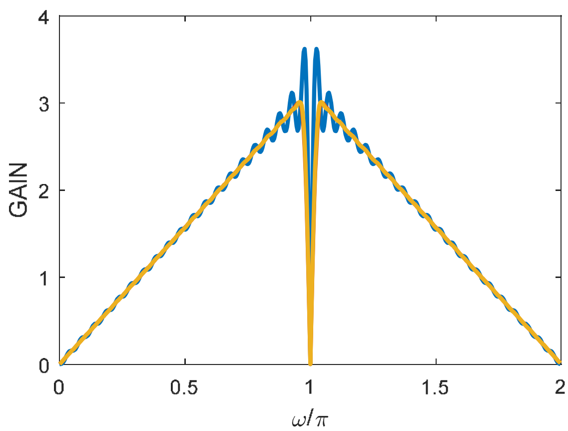
Note 1: the solution should be: $h[n] = \begin{cases} 0, & n = 0 \\ \frac{\cos(n\pi)}{n} = \frac{(-1)^n}{n}, & n \neq 0 \end{cases}$

P2P assessment: **3pt /5** if demonstration is clear and complete, and results are correct

Note 2: if a detailed explanation is not provided that explains the particular result for $h[0]$, which involves use of the L'Hôpital rule, an overall evaluation higher that 2 cannot be given.

b) Based on the ideal impulse response obtained in **a)**, use the window method of FIR filter design with two different windows: rectangular and Hamming. Obtain the corresponding impulse responses (where $n = -40, -39, \cdots, 39, 40$) and represent them graphically. Also, obtain the corresponding frequency response magnitudes, represent them graphically, and explain to your colleagues what the main differences are.

Note: The combined graphical illustration should look like:



P2P assessment: **1pt /5** if demonstration is clear and complete, and results are correct

c) Design an optimal FIR differentiator filter using the Parks McClellan algorithm in Matlab by using the following Matlab command (or by using the Matlab `fdatool` environment):

```
h3=firpm(order,[0 0.96],[0 0.96*pi],'differentiator');
```

Represent graphically the impulse response of this filter and compare it to the impulse responses of the previous two designs. In addition, represent graphically all three frequency response magnitudes and highlight the main differences.

P2P assessment: **1pt /5** if demonstration is clear and complete, and results are correct

d) Explain to your colleagues what type of FIR linear-phase filter the discrete-time differentiator corresponds to, and what is the realization structure that allows its efficient implementation. Indicate what the implementation cost is in terms of memory (for data and filter coefficients) and arithmetic operations (i.e., multiplications and additions).

e) Explain to your colleagues what the discrete-time differentiator is useful for, and provide at least one application example.

P2P Exercise 2

In this exercise, we design a linear-phase FIR Hilbert Transformer of order 80 (i.e., the length of the impulse response is 81 samples) whose ideal 2π–periodic frequency response is specified as follows:

$$H\left(e^{j\omega}\right) = \begin{cases} j, & -\pi \le \omega < 0 \\ -j, & 0 \le \omega < \pi \end{cases}$$

a) Find the impulse response and explain to your colleagues how you obtain it.

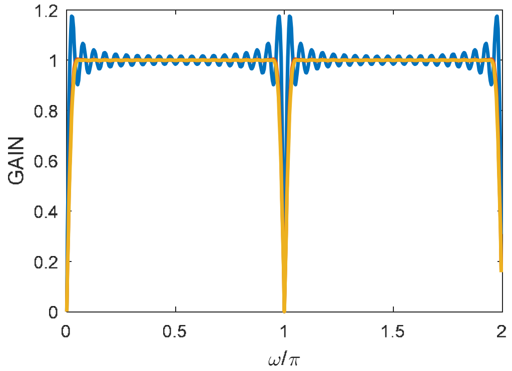
Note 1: the solution should be: $h[n] = \begin{cases} 0, & n = 0 \\ \frac{1-\cos(n\pi)}{n\pi} = 2\frac{(\sin(n\pi/2))^2}{n\pi}, & n \neq 0 \end{cases}$

P2P assessment: **3pt /5** if demonstration is clear and complete, and results are correct

Note 2: if a detailed explanation is not provided that explains the particular result for $h[0]$, which involves use of the L'Hôpital rule, an overall evaluation higher that 2 cannot be given.

b) Based on the ideal impulse response obtained in **a)**, use the window method of FIR filter design with two different windows: rectangular and Hamming. Obtain the corresponding impulse responses (where $n = -40, -39, \cdots, 39, 40$) and represent them graphically. Also, obtain the corresponding frequency response magnitudes, represent them graphically, and explain to your colleagues what the main differences are.

Note: Regarding frequency response magnitude, the combined graphical illustration should look like:



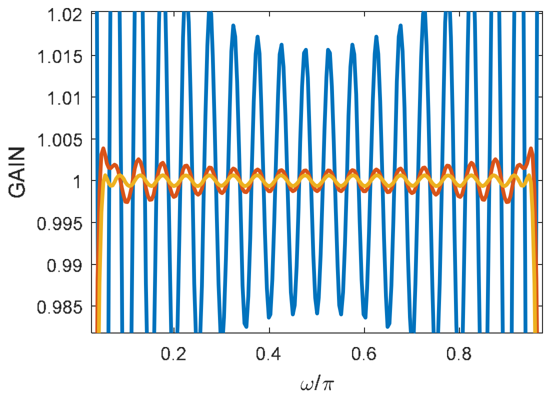
P2P assessment: **1pt /5** if demonstration is clear and complete, and results are correct

c) Design an optimal FIR Hilbert Transformer using the Parks McClellan algorithm in Matlab by using the following Matlab command (or by using the Matlab `fdatool` environment):

```
h3=firpm(order,[0.05 0.95],[1 1],'Hilbert');
```

Represent graphically the impulse response of this filter and compare it with the impulse responses of the previous two designs. In addition, represent graphically all three frequency response magnitudes and highlight the main differences.

Note: You should zoom-in the frequency response magnitudes in the pass-band so as to highlight the associated ripples, as illustrated next:



P2P assessment: **1pt /5** if demonstration is clear and complete, and results are correct

d) Explain to your colleagues what type of FIR linear-phase filter the Hilbert Transformer corresponds to, and what is the realization structure that allows its efficient implementation. Indicate what the implementation cost is in terms of memory (for data and filter coefficients) and arithmetic operations (i.e., multiplications and additions).

e) Explain to your colleagues what the Hilbert Transformer is useful for, and provide at least one application example.