L.EEC025 - FUNDAMENTALS OF SIGNAL PROCESSING

Academic year 2024-2025, week 7
P2P exercises

Exercises related to "Peer-to-peer learning/assessment" (P2P L/A)

Preliminary considerations

The P2P exercises this week articulate with the lab (PL) class experimental objectives. The P2P exercises are meant to motivate a theoretical analysis of comb filters, especially regarding their frequency response magnitude. The lab class is meant to motive the practical measurement of the frequency response magnitude of two comb filters so as to enable a comparison between experimental results and theoretical results.

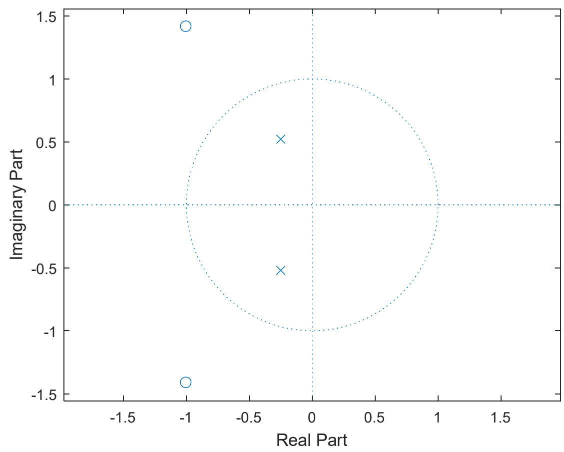
In both of the P2P exercises we use three Matlab functions: `roots()`, `zplane()`, and `freqz()`.

Regarding `roots()`: it computes the roots of a polynomial, for example, if that polynomial is $1 + 2z^{-1} + 3z^{-2}$, then, in Matlab, the roots are computed using `b=[1 2 3]; roots(b)`

Note that the inverse of `roots()` is `poly()`, thus, `poly(roots([1 2 3]))` delivers `1, 2, 3`.

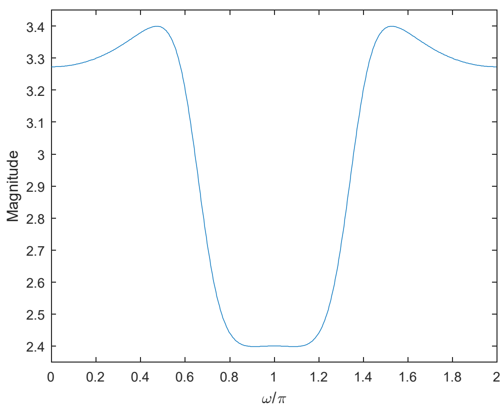
Regarding `zplane()`: it finds and represents the zeros (using the "o" symbol) and the poles (using the "x" symbol) of a transfer function on the finite Z plane, for example, if the transfer function is $\frac{1+2z^{-1}+3z^{-2}}{1+(1/2)z^{-1}+(1/3)z^{-2}}$, then the following Matlab code generates the Z-plane representation on the right:

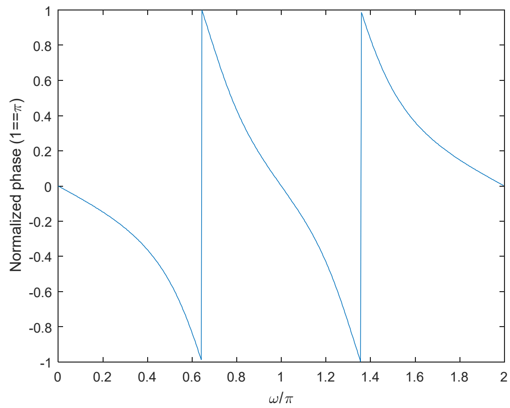
```
b=[1 2 3];
a=[1 1/2 1/3];
zplane(b,a)
```



Regarding `freqz()`: it computes the complex frequency response of a discrete-time system given its transfer function specification, for example, if the transfer function is $\frac{1+2z^{-1}+3z^{-2}}{1+(1/2)z^{-1}+(1/3)z^{-2}}$, and assuming causality, then the magnitude part of the frequency response, and the phase part of the frequency response can be computed and represented as follows:

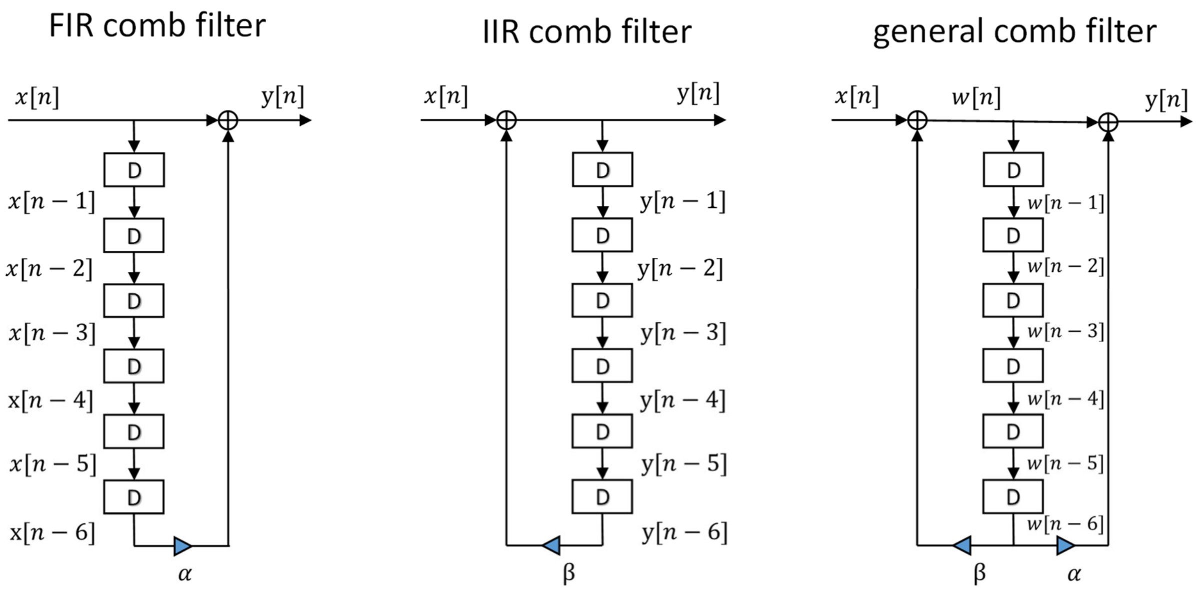
```
b=[1 2 3];
a=[1 1/2 1/3];
[H, W]=freqz(b,a,'whole');
figure(1); plot(W/pi, abs(H)); xlabel('\omega/\pi'); ylabel('Magnitude');
figure(2); plot(W/pi, angle(H)/pi); xlabel('\omega/\pi');
ylabel('Normalized phase (1==\pi)');
```





We provide here a brief introduction to comb filters.

As in the case of the moving average filter, the comb filter is a very simple type of filter that has peculiar characteristics which make it very convenient and useful in a number of applications including special audio effects, digital interpolation and digital decimation. The basic structure of a comb filter consists of the simple combination of an input signal (x[n]) and a delayed and scaled version of the input signal, or of the output signal (y[n]), or both. Thus, comb filters have three types of structure: an FIR structure, an IIR structure, or a combined FIR-IIR structure that we call general structure. These are illustrated in the following figure. The examples that are illustrated in this figure assume that the delays affecting either the input sequence, or the output sequence, involve 6 samples. This is also the scenario that we assume in the lab experiments.



Using the Z-Transform and its properties, and assuming causality, it can be easily shown that the transfer function of the illustrated FIR comb filter is

$$\frac{Y(z)}{X(z)} = 1 + \alpha z^{-6}, \ |z| > 0, \qquad (1)$$

the transfer function of the illustrated IIR comb filter is

$$\frac{Y(z)}{X(z)} = \frac{1}{1-\beta z^{-6}}, \ |z| > \sqrt[6]{|\beta|}, \qquad (2)$$

and that the transfer function of the illustrated general comb filter is

$$\frac{Y(z)}{X(z)} = \frac{1+\alpha z^{-6}}{1-\beta z^{-6}}, \ |z| > \sqrt[6]{|\beta|}. \qquad (3)$$

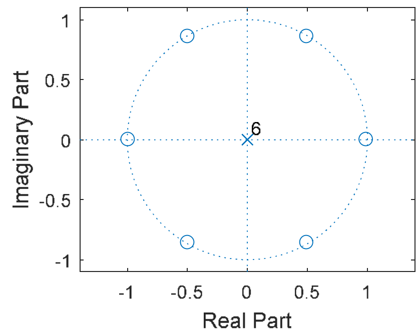
The peculiar characteristics of comb filters that justify their designation are manifested in the frequency domain, specifically, by the shape of the magnitude frequency response. As it is assumed in the lab (PL) class experiments, we also admit in both P2P exercises that $\alpha = -0.95$, and $\beta = -0.6$.

P2P Exercise 1

In this exercise, we analyse the FIR comb filter as specified above.

a) Find (by hand) the zeros and the poles of the transfer function of the FIR comb filter. Use the `roots()` Matlab function to validate your analytical results and use the `zplane()` Matlab command to represent the poles and zeros of the FIR comb filter transfer function.

Note: the result of the `zplane()` Matlab command should be:

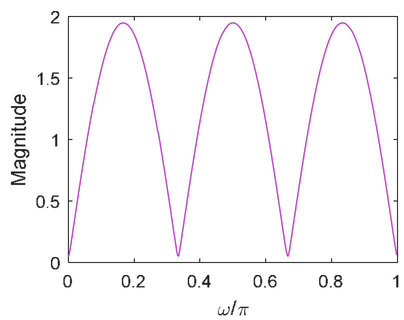


P2P assessment: **2pt /5** if analytical results and graphical representation are correct

b) Find a compact (real-valued) expression describing the magnitude frequency response of the FIR comb filter. Complete and use the following Matlab code to overlap the graphical representation of this real-valued expression to that delivered by the `freqz()` Matlab command:

```
grid=[0:1/512:1-1/512];
b = to be completed
[H, W]=freqz(b,1);
plot(W/pi, abs(H))
HFIR = to be completed
pause
hold on
plot(grid, HFIR, 'm')
hold off
```

Note: the solution should be: $|H(e^{j\omega})| = \sqrt{1 + 2\alpha\cos(6\omega) + \alpha^2}$, and the graphical representation should be



P2P assessment: **2pt /5** if analytical results and graphical representation are correct

c) Show that $\max_{\omega}|H(e^{j\omega})| = |1 + |\alpha||$, and that $\min_{\omega}|H(e^{j\omega})| = |1 - |\alpha||$

P2P assessment: **1pt /5** if demonstration is correct

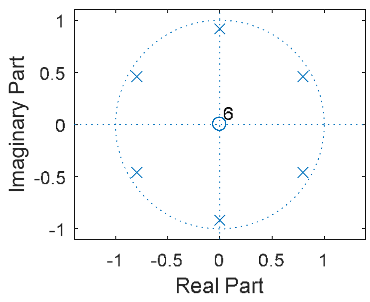
d) Assuming that the sampling frequency is 8 kHz, when the frequency response of the FIR comb filter is experimentally measured in the lab, how many peaks should you expect to observe in the magnitude frequency response and at what frequencies (in Hz) ? And how many valleys and at what frequencies (in Hz) ?

P2P Exercise 2

In this exercise, we analyse the IIR comb filter as specified above.

a) Find (by hand) the zeros and the poles of the transfer function of the IIR comb filter. Use the `roots()` Matlab function to validate your analytical results and use the `zplane()` Matlab command to represent the poles and zeros of the IIR comb filter transfer function.

Note: the result of the `zplane()` Matlab command should be:

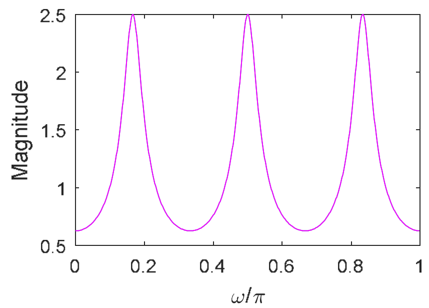


P2P assessment: **2pt /5** if analytical results and graphical representation are correct

b) Find a compact (real-valued) expression describing the magnitude frequency response of the IIR comb filter. Complete and use the following Matlab code to overlap the graphical representation of this real-valued expression to that delivered by the `freqz()` Matlab command:

```
grid=[0:1/512:1-1/512];
a = to be completed
[H, W]=freqz(1,a);
plot(W/pi, abs(H))
HIIR = to be completed
pause
hold on
plot(grid, HIIR, 'm')
hold off
```

Note: the solution should be: $\left|H\left(e^{j\omega}\right)\right| = \frac{1}{\sqrt{1-2\beta\cos(6\omega)+\beta^2}}$



P2P assessment: **2pt /5** if analytical results and graphical representation are correct

c) Show that $\max_{\omega}\left|H\left(e^{j\omega}\right)\right| = \frac{1}{|1-|\beta||}$, and that $\min_{\omega}\left|H\left(e^{j\omega}\right)\right| = \frac{1}{|1+|\beta||}$

P2P assessment: **1pt /5** if answer is correct

d) Assuming that the sampling frequency is 8 kHz, when the frequency response of the IIR comb filter is experimentally measured in the lab, how many peaks should you expect to observe in the magnitude frequency response and at what frequencies (in Hz) ? And how many valleys and at what frequencies (in Hz) ?

Extra question to be addressed by all group members: in the case of the general comb filter, as above, how many peaks should you expect to observe in the magnitude frequency response ? In what sense do these peaks differ from those that emerge from P2P Exercise 1, or P2P Exercise 2 ?